1. NUMBER SYSTEM & SIMPLIFICATION

The ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called digits, which can represent any number.

Natural Numbers: These are the numbers (1, 2, 3, etc.) that are used for counting. It is denoted by N.

There are infinite natural numbers and the smallest natural number is one (1)

Even numbers: Natural numbers which are divisible by 2 are even numbers. It is denoted by E.

E = 2, 4, 6, 8, ...

Smallest even number is 2. There is no largest even number.

Odd numbers: Natural numbers which are not divisible by 2 are odd numbers.

It is denoted by O.

O = 1, 3, 5, 7, ...

Smallest odd number is 1.

There is no largest odd number.

Based on divisibility, there could be two types of natural numbers:

Prime and Composite.

a) Prime Numbers: Natural numbers which have exactly two factors, i.e., 1 and the number itself are called prime numbers. The lowest prime number is 2. 2 is also the only even prime number.

b) Composite Numbers: It is a natural number that has atleast one divisor different from unity and itself.

Every composite number can be factorized into its prime factors.

For example: 24 = 2 × 2 × 2 × 3. Hence, 24 is a composite number.

The smallest composite number is 4.

Whole Numbers: The natural numbers along with zero (0), from the system of whole numbers.

It is denoted by W.

There is no largest whole number and

The smallest whole number is 0.

Integers: The number system consisting of natural numbers, their negative and zero is called integers.

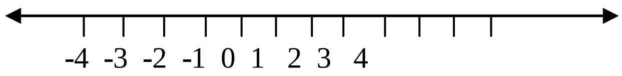
It is denoted by Z or I.

The smallest and the largest integers cannot be determined.

Remember

- ❖ 1 is neither prime nor composite
- ❖ 1 is an odd integer.
- ❖ 0 is neither positive nor negative.
- ❖ 0 is an even integer.
- ❖ 2 is prime & even both.
- ❖ All prime numbers (except 2) are odd.

The number line: The number line is a straight line between negative infinity on the left to positive infinity on the right.



Real Numbers: All numbers that can be represented on the number line are called real numbers.

It is denoted by R.

R^+: Positive real numbers and

R^- : Negative real numbers.

Real numbers = Rational numbers + Irrational numbers.

a) **Rational numbers:** Any number that can be put in the form of $\frac{p}{q}$, where p and q are integers and q ≠ 0, is called a rational number.

It is denoted by Q.

Every integer is a rational number.

Zero (0) is also a rational number. The smallest and largest rational numbers cannot be determined. Every fraction (and decimal fraction) is a rational number.

$$Q = \frac{\text{p (Numerator)}}{\text{q (Denominator)}}$$

❖ If x and y are two rational numbers, then $\frac{x+y}{2}$ is also a rational number and its value lies between the given two rational numbers x and y.

❖ An infinite number of rational number can be determined between any two rational numbers.

Example 1:

Find three rational numbers between 3 and 5.

Solution:

1^{st} rational number $= \frac{3+5}{2} = \frac{8}{2} = 4$

2^{nd} rational number (i.e., between 3 and 4)

$= \frac{3+4}{2} = \frac{7}{2}$

3^{rd} rational number (i.e., between 4 and 5)

$= \frac{4+5}{2} = \frac{9}{2}$

b) **Irrational numbers:** The numbers which are not rational or which cannot be put in the form of $\frac{p}{q}$, where p and q are integers and q ≠ 0, is called irrational number.

It is denoted by Q' or Q^c.

$\sqrt{2}, \sqrt{3}, \sqrt{5}, 2+\sqrt{3}, 3-\sqrt{5}, 3\sqrt{3}$ are irrational numbers.

NOTE:

(i) Every positive irrational number has a negative irrational number corresponding to it.

(ii) $\sqrt{2}+\sqrt{3} \neq 5$

$$\sqrt{5}-\sqrt{3} \neq \sqrt{2}$$

$$\sqrt{3}\times\sqrt{2} = \sqrt{3\times 2} = \sqrt{6}$$

$$\sqrt{6}\div\sqrt{2} = \sqrt{\frac{6}{2}} = \sqrt{3}$$

(iii) Some times, product of two irrational numbers is a rational number.

For example: $\sqrt{2}\times\sqrt{2} = \sqrt{2\times 2} = 2$

$(2+\sqrt{3})\times(2-\sqrt{3}) = (2)^2 - (\sqrt{3})^2$=4-3=1

❖ Both rational and irrational numbers can be represented on number line. Thus real numbers is the set of the union of rational and irrational numbers.

R = Q ∪ Q′

❖ Every real numbers is either rational or irrational.

Fraction: A fraction is a quantity which expresses a part of the whole.

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$

Example 2:

Write a fraction whose numerator is $2^2 + 1$ and denominator is $3^2 - 1$.

Solution:

Numerator = $2^2 + 1 = 4 + 1 = 5$

Denominator = $3^2 - 1 = 9 - 1 = 8$

$\therefore$ Fraction $= \frac{\text{Numerator}}{\text{Denominator}} = \frac{5}{8}$

TYPES OF FRACTIONS:

a) **Proper fraction :** If numerator is less than its denominator, then it is a proper fraction:

For example: $\frac{2}{5}, \frac{6}{18}$

b) **Improper fraction:** If numerator is greater than or equal to its denominator, then it is a improper fraction.

For example: $\frac{5}{2}, \frac{18}{7}, \frac{13}{13}$

NOTE:

If in a fraction, its numerator and denominator are of equal value then fraction is equal to unity i.e.1.

c) **Mixed fraction:** it consists of an integer and a proper fraction.

For example: $1\frac{1}{2}$, $3\frac{2}{3}, 7\frac{5}{9}$

NOTE:

Mixed fraction can always be changed into improper fraction and vice versa.

For example: $7\frac{5}{9} = \frac{7\times 9+5}{9} = \frac{63+5}{9} = \frac{68}{9}$

and $\frac{19}{2} = \frac{9\times 2+1}{2} = 9 + \frac{1}{2} = 9\frac{1}{2}$

d) **Equivalent fraction/Equal fractions:** Fractions with same value.

For example: $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}\left(\frac{2}{3}\right)$

e) **Like fractions:** Fractions with same denominators.

For example: $\frac{2}{5}, \frac{3}{7}, \frac{9}{8}, \frac{11}{16}$

f) **Unlike fractions:** Fractions with different denominators.

For example: $\frac{2}{5}, \frac{4}{7}, \frac{9}{8}, \frac{9}{2}$

NOTE:

Unlike fractions can be converted into like fractions.

For example: $\frac{3}{5}\ and\ \frac{4}{7}$

$\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$ and $\frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$

g) **Simple fractions:** Numerator and denominator are integers.

For example: $\frac{3}{7}\ and\ \frac{2}{5}$

h) **Complex fraction:** Numerator or denominator or both are fractional numbers.

For example: $\frac{\frac{2}{5}}{7}, \frac{2\frac{1}{3}}{5\frac{2}{3}}, \frac{2+\frac{1+\frac{2}{7}}{3}}{2}$

i) **Decimal fraction:** Denominator with the powers of 10.

For example: $\frac{2}{10} = (0.2), \frac{9}{100} = (0.09)$

j) **Vulgar fraction:** Denominators are not the power of 10.

For example: $\frac{3}{7}, \frac{9}{2}, \frac{5}{193}$.

Example 3:

After doing 3/5 of the Biology homework on Monday night, Sanjay did 1/3 of the remaining homework on Tuesday night. What fraction of the original homework would Sanjay have to do on Wednesday night to complete the Biology assignment?

Solution:

Remaining homework on Monday night

$= 1 - \frac{3}{5} = \frac{2}{5}$

Work done on Tuesday night $= \frac{1}{3}$ of $\frac{2}{5} = \frac{2}{15}$

Remaining homework to complete the biology assignment

$= \frac{2}{5} - \frac{2}{15} = \frac{6-2}{15} = \frac{4}{15}$

Rounding off (Approximation) of Decimals: There are some decimals in which numbers are found upto large number of decimal places.

For example: 3.4578, 21.358940789.

But many times we require decimal numbers upto a certain number of decimal places. Therefore,

If the digit of the decimal place is five or more than five, then the digit in the preceding decimal place is increased by one and if the digit in the last place is less than five, then the digit in the precedence place remains unchanged.

Example 4:

(a) Write 21.3751 upto two places of decimal.

(b) Write 3.27645 upto three places of decimal.

Solution:

(a) 21.3751 = 21.38

(b) 3.27645 = 3.276

Operations: The following operations of addition, subtraction, multiplication and division are valid for real numbers.

(a) Commutative property of addition:
a + b = b + a

(b) Associative property of addition:
(a + b) + c = a + (b + c)

(c) Commutative property of multiplication:
a * b = b * a

(d) Associative property of multiplication:
(a * b) * c = a * (b * c)

(e) Distributive property of multiplication with respect to addition:
(a + b) * c = a * c + b * c

Complex numbers: A number of the form a + bi, where a and b are real number and i = $\sqrt{-1}$ (imaginary number) is called a complex number.

It is denoted by C.

For Example: 5i (a = 0 and b = 5), $\sqrt{5}$ + 3i (a = $\sqrt{5}$ and b = 3)

NOTE:

$i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

DIVISIBILITY RULES

Divisibility by 2: A number is divisible by 2 if it's unit digit is even or 0.

Divisibility by 3: A number is divisible by 3 if the sum of it's digit are divisible by 3.

Divisibility by 4: A number is divisible by 4 if the last 2 digits are divisible by 4, or if the last two digits are 0's.

Divisibility by 5: A number is divisible by 5 if it's unit digit is 5 or 0.

Divisibility by 6: A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisibility by 7: We use osculator (-2) for divisibility test.

99995: 9999 – 2 × 5 = 9989
9989: 998 – 2 × 9 = 980
980: 98 – 2 × 0 = 98

Now 98 is divisible by 7, so 99995 is also divisible by 7.

Divisible by 11: In a number, if difference of sum of digit at even places and sum of digit at odd places is either 0 or multiple of 11, then no. is divisible by 11.
For example, 12342 ÷ 11
Sum of even place digit = 2 + 4 =6
Sum of odd place digit = 1 + 3 + 2= 6
Difference = 6 – 6 = 0
∴ 12342 is divisible by 11.

Divisible by 13: we use (+4) as osculator.
e.g., 876538 ÷ 13

876538: 8 × 4 + 3 = 35
5 × 4 + 3 + 5 = 28
8 × 4 + 2 + 6 = 40
0 × 4 + 4 + 7 = 11
1 × 4 + 1 + 8 = 13
13 is divisible by 13.
∴ 876538 is also divisible by 13.

Divisible by 17: We use (-5) as osculator.

e.g., 294678: 29467 – 5 × 8 = 29427
27427: 2942 – 5 × 7 =2907
2907: 290 – 5 × 7 = 255
255: 25 – 5 × 5 =0
∴ 294678 is completely divisible by 17.

Divisible by 19: We use (+2) as osculator.

e.g., 149264: 4 × 2 + 6 = 14

4 × 2 +1 + 2 = 11

1 × 2 + 1 + 9 =12

2 × 2 + 1 + 4 = 9

9 × 2 +1 = 19

19 is divisible by 19

∴ 149264 is divisible by 19.

Divisibility by a Composite number:

A number is divisible by a given composite number if it is divisible by all factors of composite number.

Example 5:

Is 2331024 divisible by 12

Solution:

12 = 4 × 3

2331024 is divisible by 3 as (2+3+3+1+2+4)= 15 is divisible by 3 2331024 is also divisible by 4 because last two digits (24) is divisible by 4

Therefore 2331024 is divisible by 12

Example 6:

What is the value of M and N respectively if M39048458N is divisible by 8 and 11, where M and N are single digit integers?

Solution:

A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

i.e., 58N is divisible by 8.

Clearly, N = 4

Again, a number is divisible by 11 if the difference between the sum of digits at even places and sum of digits at the odd places is either 0 or is divisible by 11.

i.e. (M+9+4+4+8) – (3+0+8+5+N)

= M + 25 – (16 + N)

= M – N + 9 must be zero or it must be divisible by 11

i.e. M – N = 2

⇒ M = 2 + 4 = 6

Hence, M = 6; N = 4

Example 7:

The highest power of 9 dividing 99! completely is:

Solution:

(c) $9 = 3 \times 3 = 3^2$

Highest power of 3 in 99!

$= \left[\frac{99}{3}\right] + \left[\frac{99}{3^2}\right] + \left[\frac{99}{3^3}\right] + \left[\frac{99}{3^4}\right]$

= 33 + 11 + 3 +1 = 48

But we have 3^2

highest power of 9 in 99! $= \frac{48}{2} = 24$

DIVISION ALGORITHM:

Dividend = (Divisor × Quotient) + Remainder where, Dividend = The number which is being divided Divisor = The number which performs the division process Quotient = Greatest possible integer as a result of division Remainder = Rest part of dividend which cannot be further divided by the divisor.

Complete remainder: A complete remainder is the remainder obtained by a number by the method of successive division.

Complete reminder = [I divisor × II remainder] + I remainder

$$\boxed{\begin{aligned} &C.R. = d_1 r_2 + r_1 \\ &C.R. = d_1 d_2 r_3 + d_1 r_2 + r_1 \end{aligned}}$$

❖ Two different numbers x and y when divided by a certain divisor D leave remainder r_1 and r_2 respectively. When the sum of them is divided by the same divisor, the remainder is r_3. Then,

$$\boxed{\text{divisor } D = r_1 + r_2 - r_3}$$

Method to find the number of different divisors (or factors) (including 1 and itself) of any composite number N:

STEP I: Express N as a product of prime numbers as.

$N = x^a \times y^b \times z^c$

STEP II: Number of different divisors (including l and itself)

= (a + 1) (b + 1) (c +1) .....

Example 8:

Find the number of different divisors of 50, besides unity and the number itself.

Solution:

If you solve this problem without knowing the rule, you will take the numbers in succession and check the divisibility. In doing so, you may miss some numbers. It will also take more time.

Different divisors of 50are: 1,2,5,10,25,50

If we exclude 1 and 50, the number of divisors will be 4.

By rule: 50 -2 × 5 × 5 = $2^1 \times 5^2$

∴the number of total divisors = (l + 1) × (2+1) = 2 × 3 = 6 or, the number of divisors excluding 1 and 50 = 6 – 2 = 4

Example 9:

A certain number when divided by 899 leaves the remainder 63. Find the remainder when the same number is divided by 29.

Solution:

Number = 899Q+63, where Q is quotient

= 31 × 29Q+ (58 + 5) =29[31Q+2] + 5

∴Remainder = 5

Counting Number of Zeros

Sometimes we come across problems in which we have to count number of zeros at the end of factorial of any numbers. for example- Number of zeros at the end of 10!

10! = 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 ×1

Here basically we have to count number of fives, because multiplication of five by any even number will result in 0 at the end of final product. In 10! we have 2 fives thus total number of zeros are 2.

Short cut:-

Counting number of zeros at the end of n! value will be

$$\frac{n}{5}+\frac{n}{5^2}+\frac{n}{5^3}+\frac{n}{5^4}+\ldots\ldots$$

The integral value of this number will be the total number of zeros.

Example 10:

Number of zeros at the end of 10!

Solution: $\frac{10}{5}+\frac{10}{5^2}+\ldots\ldots$ Integral value

= 2 + 0

So, number of zeros in 10! = 2.

Note:- Here $\frac{10}{5^2}$ is less than 1 so will not count it.

Example 11:

Number of zeros at the end of 100!

Solution: $\frac{100}{5}+\frac{100}{5^2}+\frac{100}{5^3}+\ldots\ldots$

integral value will be

= 20+4 = 24 zeros.

Example 12:

Number of zeros at the end of 126!

Solution:

$\frac{126}{5}+\frac{126}{5^2}+\frac{126}{5^3}+\ldots\ldots$

⇒integral value will be

= 25 + 5 + 1 = 31 zeros.

Example 13:

Number of zeros at the end of90!

Solution:

$\frac{90}{5}+\frac{90}{5^2}+\frac{90}{5^3}+\ldots\ldots = 18 + 3 = 21$ zeros

Power of a number contained in a factorial

Highest power of a prime number P in N!

$=\left[\frac{N}{P}\right]+\left[\frac{N}{P^2}\right]+\left[\frac{N}{P^3}\right]+\cdots+\left[\frac{N}{P^r}\right]$ where [x] denotes the greatest integers less than or equal to x and is a natural number such that $P^r < n$.

Example 14:

Find highest power of 7^n in 50!

Solution:

The highest power 7 in 50!

$=\left[\frac{50}{7}\right]+\left[\frac{50}{7^2}\right] = 7 + 1 = 8$

Example 15:

Find highest power 15 in 100!

Solution:

Here given number 15 is not a prime number so first convert 15 as product of Primes 15= 3× 5 therefore we will find the highest power of 3 and 5 in 1001 Highest power of 3 in 100!

$=\left[\frac{100}{3}\right]+\left[\frac{100}{3^2}\right]+\left[\frac{100}{3^3}\right]+\frac{100}{3^4}]$

= 33 +11 + 3 + 1 = 48

Highest power of 5 in 100!

$= \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] = 20+4 = 24$

So 100! contains $(3)^{48} \times (5)^{24}$. Hence it contains 24 pairs of3 and5.Therefore, required power of 15 is 24, which is actually the power of the largest prime factor 5 of 15, because power of largest prime factor is away equal to or less than the other prime factor of any number.

TO FIND THE LAST DIGIT OR DIGIT ATTHE UNIT'S PLACE OF a^n.

(i) If the last digit or digit at the unit's place of a is 1,5 or 6, whatever be the value of n, it will have the same digit at unit's place, i.e,

$(.....1)^n = (......1)$

$(.....5)^n = (......5)$

$(.....6)^n = (......6)$

(ii) If the last digit or digit at the units place of a is 2,3,5,7 or 8, then the last digit of an depends upon the value of n and follows a repeating pattern in terms of 4 as given below:

n	last digit of $(....2)^n$	last digit of $(....3)^n$	last digit of $(....7)^n$	last digit of $(....8)^n$
4x + 1	2	3	7	8
4x + 2	4	9	9	4
4x + 3	8	7	3	2
4x	6	1	1	6

(iii) If the last digit or digit at the unit's place of a is 4 or 9, then the last digit of a^n depends upon the value of n and follows repeating pattern in terms of2 as given below.

n	Last digit of $(...4)^n$	Last digit of $(...9)^n$
2x	6	1
2x+1	4	9

Example 16:

Find unit digit of2^{323}.

Solution: Here, 2,4,8,6 will repeat after every four interval till 320 next digit will be 2,4,$\boxed{8}$, so unit digit of 2^{323} will be 8.

Example 17:

Find unit digit of 133^{133}.

Solution:

Cycle of 3 is 3,9,7,1 which repeats after every fourth interval will 133^{132}, so next unit digit will be 3.

Example 18:

Find unitdigitof$963^{63} \times 73^{73}$.

Solution: Unit digit of963^{63} = 7

Unit digit of 73^{73}=3

So unit digit of$963^{63} \times 73^{73} = 7 \times 3 = 21$.

i.e. 1.

Example 19:

Find Unit digit of $17^{17} \times 27^{27} \times 37^{37}$

Solution: Unit digit of 17^{17} = 7

Unit digit of27^{27} = 3

Unit digit of37^{37}= 7

So unit digit of $17^{17}\times 27^{27}\times 37^{37}$=7 × 3 × 7 =147

i.e., unit digit = 7

Example 20:

Find unit digit of $18^{18}\times 28^{28}\times 288^{288}$.

Solution:

Unit digit of 18^{18} is 4.

Unit digit of 28^{28} is 6.

Unit digit of 288^{288} is 6

So unit digit of $18^{18}\times 28^{28}\times 288^{288}$

= 4 × 6 × 6 = 144 i.e.,4

Example 21:

Find unit digit of $11^{11} + 12^{12}+13^{13}+14^{14} + 15^{15}$

Solution:

Unit digit of 11^{11} = 1

Unit digit of 12^{12} = 6

Unit digit of 13^{13} = 3

Unit digit of 14^{14} = 6

Unit digit of 15^{15} = 5

So unit digit of given sum will be 1+6+3+6+5=21 i.e., 1

Example 22:

Find unit digit of

$21^{21}\times 22^{22}\times 23^{23}\times 24^{24}\times 25^{25}$.

Solution:

25^{25} will give 5 in unit place, when multiplied by an even number i.e. 0,2,4,6,8. It will give zero at unit place. So, zero will be at the unit digit of given question.

REMAINDERTHEOREM

Remainder of expression$\frac{a\times b\times c}{n}$ [i/e. a × b × c when divided by n] is equal to the remainder of expression$\frac{a_r\times b_r\times c_r}{n}$ [i.e. $a_r\times b_r\times c_r$ when divided by n], where

a_r is remainder when a is divided by n.

b_r is remainder when b, is divided by n, and

c_r is remainder when c is divided by n.

Example 23:

Find remainder of 15 × 17 × 19 when divided by 7.

Solution:

Remainder of Expression $\frac{15\times 17\times 19}{7}$ will be equal to

$\frac{1\times 3\times 5}{7} = \frac{15}{7} = 2\frac{1}{7}$ i.e. 1

On dividing 15 by 7, we get 1 as remainder.

On dividing 17 by 7, we get 3 as remainder.

On dividing 19 by 7, we get 5 as remainder.

And combined remainder will be equal to remainder of $\frac{15}{7}$ i.e. 1.

Example 24:

Find the remainder of expression$\frac{19\times 20\times 21}{9}$

Solution:

Remainder of given expression =$\frac{1\times 2\times 3}{9}$ = $\frac{6}{9}$equal to 6.

POLYNOMIALTHEOREM

This is very powerful theorem to find the reminder.

According to polynomial theorem.

$(x+a)^n = x^n + n_{c_1}x^{n-1}a^1 + n_{c_2}x^{n-2}a^2 + n_{c_3}x^{n-3}a^3 + \cdots n_{c_{n-1}}x^1a^{n-1} + a^n$ .... (i)

$\therefore \frac{(x+a)^n}{x} = \frac{x^n + n_{c_1}x^{n-1}a^1 + n_{c_2}x^{n-2}a^2 + n_{c_3}x^{n-3}a^3 + \cdots n_{c_{n-1}}x^1a^{n-1} + a^n}{x}$

remainder of expression (ii) will be equal to remainder of$\frac{a^n}{x}$ because rest of the term contains x are completely divisible by x.

Example 25:

Find the remainder of$\frac{9^{99}}{8}$.

Solution:

$$\frac{9^{99}}{8} = \frac{(8+1)^{99}}{8}$$

According to polynomial theorem remainder will be equal to remainder of the expression $\frac{1^{99}}{8} = \frac{1}{8}, 1$

Example 26:

Find the remainder of $\frac{8^{99}}{7}$

Solution:

$\frac{8^{99}}{7} \Rightarrow \frac{(7+1)}{7} = \frac{1^{99}}{7}$ i.e. 1

Example 27:

Find remainder of $\frac{11\times13\times17}{6}$

Solution:

$$\frac{11 \times 13 \times 17}{6} = \frac{5 \times 1 \times 5}{6}$$

$$\frac{1}{6} \Rightarrow 1$$

Example 28:

Find remainder of $\frac{9^{100}}{7}$.

Solution:

$\frac{9^{100}}{7} \Rightarrow \frac{(7+2)^{100}}{7}$

$= \frac{2^{100}}{7} = \frac{2^{99}\times2}{7} = \frac{2^{3\times33}\times2}{7} = \frac{8^{33}\times2}{7}$

$= \frac{(7+1)^{33}}{7} \times 2 = \frac{1\times2}{7} = \frac{2}{7}$ i.e. 2

Example 29:

Find remainder of $\frac{9^{50}}{7}$.

Solution:

$\frac{9^{50}}{7} = \frac{(7+2)^{50}}{7} = \frac{2^{50}}{7} = \frac{(2^3)^{16}\times2^2}{7} = \frac{8^{16}\times4}{7}$

$\Rightarrow \frac{(7+1)^{16}\times4}{7} = \frac{1\times4}{7}$ i.e., 4

Example 30:

Find remainder of $\frac{25^{100}}{7}$.

Solution:

$\frac{25^{100}}{7} = \frac{(3\times7+4)^{50}}{7} = \frac{4^{50}}{7}$

$\frac{2^{100}}{7} = \frac{(2^3)^{33}\times2}{7} \Rightarrow \frac{(7+1)^{33}}{7} \times 2 \Rightarrow \frac{1\times2}{7}$

$\Rightarrow$Reminder is 2.

Example 31:

Find remainder of $\frac{3^{50}}{7}$.

Solution:

$\frac{3^{50}}{7} = \frac{(3^2)^{25}}{7} \Rightarrow \frac{(7+2)^{25}}{7}$

$= \frac{2^{25}}{7} = \frac{\left(2^3\right)^8\times2}{7}$

$= \frac{(7+1)^8\times2}{7} = \frac{1\times2}{7}$

$\Rightarrow$Reminder is 2.

Example 32:

Find remainder of $\frac{3^{250}}{7}$.

Solution:

$\frac{(3^2)^{125}}{7} = \frac{(7+2)^{125}}{7} = \frac{2^{125}}{7}$

$= \frac{\left(2^3\right)^{41}\times2^2}{7} = \frac{1\times4}{7}$

$\Rightarrow$Reminder is 4.

LAW OF SURDS

- $\left(\frac{1}{a^n}\right)^n = a$
- $\frac{1}{a^n}\frac{1}{b^n} = (ab)^{\frac{1}{n}}$
- $\left(\frac{1}{a^n}\right)^{\frac{1}{m}} = a^{\frac{1}{mn}}$

LAW OF INDICES

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $\frac{1}{a^m} = \sqrt[m]{a}$

- $a^{-m} = \frac{1}{a^m}$
- $a^{m/n} = \sqrt[n]{a^m}$
- $a^0 = 1$

Addition and subtraction of Surds

Example: $5\sqrt{2} + 20\sqrt{2} - 3\sqrt{2} = 22\sqrt{2}$

Example: $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5} = 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5} = \sqrt{5}$